

The significance of rota representation in the design of rotas

by Johannes Gärtner, Dr,¹ Sabine Wahl, DI²

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Shift scheduling is based on some representation of shift schedules. The number of different representations currently used is high, and one might expect small practical differences between these representations. However, an analysis of several prominent representations revealed strong differences regarding possible outcomes of the scheduling process and the effort needed for their assessment. Limitations of some representations do not only concern specific rota design issues, such as different staffing levels or different workhours, but also rather simple and straightforward rotas. Furthermore, there is no single representation that is strictly and unequivocally better than the others. Most representations simplify the development of some rotas, while they make it very difficult or even impossible to develop others. Therefore, both designers and the computer systems used for design should use and support several representations and therefore allow smooth transitions between them. In addition knowledge about rota construction techniques should be maintained, as it may ease assessment dramatically.

Key terms assessment of shift rotas, complexity analysis, computer-supported shift scheduling, shift scheduling.

The design, evaluation, and implementation of shift rotas are important issues in improving the work conditions of shift workers. Several guidelines exist for the design. (See, for example, reference 1.) In addition, several computer systems aim at easing the design (2—5). This article deals with a fundamental, but seldom discussed, issue of shift rota design, namely, the representation of shift rotas, that is, how they are recorded.

Many representations exist (6, 7). One might expect small differences in their relevance, however. We chose 5 representations that are — in our experience — broadly used for the design of rotas based on a small number of different shifts. In this paper we show that these 5 representations differ substantially in at least 2 dimensions.

First, we analyzed whether each representation is capable of representing all possible rotas. We started from the general definition that rotas are functions that designate duties to employees. We found severe restrictions in prominent representations. These restrictions are often advantageous as they reduce the “search space”. However, they may also oppress interesting solutions.

Second, we examined how much effort it takes to assess rotas. The representations differed strongly with respect to specific features of the rotas. Depending on the focus of the design (eg, improving weekends, optimizing rhythms, considering legal constraints) and

depending on the rota at hand, the difference is often not gradual but varies by multiples. Considering the time constraints of actual rota development, this type of variation again has practical impact. With suboptimal representations, fewer rotas can be assessed or the assessment is shallower.

Typically, representations are not self-explanatory. They build upon the knowledge of readers to interpret them. Therefore, in the first section, we introduce some important representations and briefly outline their “reading instructions”. At least to our knowledge, no well established names are in use for these representations. Therefore, we introduce names for them. A discussion of important theoretical and practical limits of a few prominent and widely used representations follows. In the 3rd section we discuss the effort needed to assess a rota in different representations.

Important representations

The “classical” representation

Prominent representations used by the night- and shift-work community are variants of the one used in the following example, a “traditional schedule” for 120 hours

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a week (8). We call it the “classical” representation (figure 1).

In the example presented in figure 1 there are 3 shifts: M=morning (8 hours, 0600—1400), E=evening (8 hours, 1400—2200), N=night (8 hours, 2200—0600). These kinds of shifts and the symbols “—” or “ ” for a day off are used throughout this article. The classical representation, as defined here and as typically used in rota design, shows only the 1st week for each group. After 1 week, group A continues with the arrangement shown for group B, then it continues with the arrangement shown for group C, and so forth. Often week numbers instead of group names are listed in the left column, and they represent the week within the shift cycle with the different groups starting their schedule at different weeks within the cycle.

Other representations

In figure 2 we introduce 3 different representations (basic series, extended classical, full-by-groups). The same continuous rota for 4 groups is represented.

Basic series. Basic series build up full rotas by repetition. The cycle length of the whole rota in weeks is typically its length in days (eg, 8 days $\Rightarrow$ 8 weeks' cycle length). (Exceptions regarding cycle length apply only to basic series whose lengths are multiples of 7 and to some basic series that are concatenations of shorter basic series.) Normally only short basic series are used to avoid rotas that are too long. In the example, the 4 groups start in different positions within the series with 2 days'

difference. This system leads to an identical rota with a slightly different group ordering. To get exactly the same rota, the starting order of the groups would be (group A $\Rightarrow$ M, B $\Rightarrow$ “—”, C $\Rightarrow$ N, D $\Rightarrow$ E). Feasible lengths of basic series are multiples of the number of groups (eg, for 4 groups: 4, 8, 12, etc). Basic series cannot be applied if the number of groups is a multiple of 7. A different name for the basic series is “duty cycle” (6).

Extended classical. The extended classical representation shows the first 2 weeks of the example rota for each group. After 2 weeks, group A continues with the rota of group B, then 2 weeks later with the rota of group C, and so forth. The difference from the classical representation is that the extended classical can build upon 2, 3, 4, or more weeks per row.

A similar representation is used in references 8 and 9. In summary, the classical representation could, theoretically, be extended differently. Instead of adding columns, one could add rows (eg, using 8 rows for 4 groups and letting group A start in row 1, group B start in row 3,

	Mo	Tu	We	Th	Fr	Sa	Su
A	M	M	M	M	M		
B	E	E	E	E	E		
C	N	N	N	N	N		

Figure 1. A 120-hour rota in the classical representation. (M = morning, E = evening, N = night)

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Figure 2. Three representations of the same rota for continuous work. (M = morning, E = evening, N = night)

etc). In general the number of weeks could be a multiple or even a divisor of the number of groups. If the starting points of the groups are distributed evenly over the rows, it allows the same rotas to be represented as the extended classical representation. This representation makes it more difficult to assess staffing levels than the extended classical representation does, and we do not know a company where it is used. Therefore it is not discussed hereafter.

Full-by-groups. In the full-by-groups representation the rota for each group is represented in a single row. Therefore the example rota needs columns for 8 weeks. The upper part of figure 2 shows the first 4 weeks; the lower part shows the next 4 weeks. Again, the numbers in the first row denote the week. This is one representation used by Gärtner (5).

Representation by month — hospitals

In this representation, an individual rota is developed for each person. Each rota is written in a single line. The main difference from the full-by-groups representation is that it builds upon months. Typically the rota is developed for each month separately. Amazingly, we found this representation in Austrian, German, American, and Japanese hospitals.

"Blind spots" of the classical representation

Generally speaking, the set of possible rotas can be defined as any time-stamped assignment of employees to duties. Mathematically it has the following function: rota (days, employees) $\Rightarrow$ duties.

The first analysis investigates whether all rotas can be represented in a specific representation. It leads to the question of whether a representation imposes constraints on some features of a rota. The second analysis is whether the resulting omissions ("blind spots") are practically relevant. In other words, do relevant rotas exist which cannot be covered by the representation?

The classical representation imposes constraints on the rota to be developed. First, the number of groups equals the number of weeks of the cycle length. Second, the rotas of all groups must be equal. The consequences are (i) the cycle length in weeks is equal to the number of groups; (ii) the workhours of all the groups must be equal; (iii) each group has the same number of duties, and the rhythm is identical for all [Therefore, the distribution of days off is identical. In addition the duties to be done on specific weekdays are equal for all (eg, every employee does the same number of night shifts on Sundays). Furthermore, all persons are scheduled to work for the whole cycle length.]; (iv) the staffing level of a shift on a given weekday is constant (eg, on Sunday morning).

These constraints lead to blind spots. Only a subset of the rotas possible can be represented (at least directly). Often schedulers intend the mentioned constraints. Then this representation may spare effort. Still, in some cases, sticking to the classical representation hinders the development of rotas that might be of interest.

Limited options due to the predefined cycle length

Obviously rotas that build upon a monthly basis cannot be represented in the classical representation. This limitation might not be a detriment if designers intend weekly patterns. But there do exist further limitations. The preceding example of the basic series, as well as some other rotas, cannot be represented in the classical representation without extensions. To name a few, the following interesting rotas for 5 groups for continuous work (represented as basic series) are not within the scope of the classical representation:

MMEENN - - - (length 10 days, cycle length 10 weeks)

MM-EENN - - - (length 10 days, cycle length 10 weeks)

MMM-EEE - - NNN - - - (length 15 days, cycle length 15 weeks)

If we ignore constant shift systems and assume equal staffing levels, then basic series that do not meet the following constraints cannot be represented by the classical representation: (i) the length of the basic series in days must be a factor of the length of the classical representation; (ii) the length of the basic series in days must be a multiple of the number of groups.

Given 4 groups and a continuous rota, only series with 4 or 28 days can be represented in the classical representation. Other series with lengths of 8 days, 12 days, 16 days, and so forth, that might be of interest cannot be represented in the classical representation. An extended classical representation or a full-by-groups representation is needed.

Rotas with different workhours — only partially possible

Reducing workhours is an important approach in improving rotas. General reductions in workhours are not always feasible. Still, it is often profitable for part of the work force to do fewer than standard workhours. For example, older employees may want to reduce their hours or at least their hours in shift work. Overtime or supplements may be paid in the form of reduced workhours instead of cash. Part-timers might be integrated, and persons from other departments might cover parts of a rota.

The classical representation allows the integration of part-timers (i) by building up full groups with, for example, part-timers (10) and (ii) by using separate rotas [eg, the weekend shifts being covered by different groups (8)]. These strategies may be useful. They ease the

design process from an ergonomic point of view, since the design of rotas for part-timers and full-timers differs. Still, a high number of potentially relevant rotas that might better exploit the benefits of merging full- and part-timers cannot even be investigated.

Rotas with a different distribution of shifts — not possible

Again, the development of rotas with, for example, differing numbers of night duties, duties on weekends, and the like is not possible directly. This type of system might be of interest in developing rotas for persons with specific requirements, for example, for older persons or for persons with young children. A further important restriction has to be emphasized. It is not possible to represent a rota that exploits variations (eg, a trainee working 3 weeks in the department). If the classical representation is used, several rotas that complement each other have to be developed. Whether a merged rota is better cannot be investigated.

Rotas with different staffing levels — not possible

Different staffing levels can be covered by the classical representation if they differ from day to day (eg, each Friday night only half of the employees being necessary). However, if these variations do not occur on a weekly basis, they cannot be covered directly in the representation. For example, every 3rd week there is an educational course; every 2nd week only half the employees are needed in the night shift due to maintenance; every 4th week there is a change from high load (continuous work) to semi-continuous work; and after 20 weeks of rota A, a change to rota B should be scheduled. Again, a split into several rotas that complement each other is necessary, and it is unclear whether the split leads to better rotas in all cases.

"Blind spots" of other representations

Basic series

Basic series allow more rotas than the classical representation; still they share most of the restrictions described in the last section (rotas with different workhours only partially possible and rotas with a different distribution of shifts and different staffing levels not possible). In addition, not all cycle lengths are allowed. The cycle length in weeks has to be a multiple of the number of groups. Besides, there is an even more important drawback. The typical basic series does not take advantage of weekends very well. In some cases this limitation is not much of a problem (eg, if the length of the series is not close to a multiple of 7 and the number of consecutive days off is high). But in other cases it is. For instance, the example given in figure 2 has only 1 weekend out of 8 completely off.

Extended classical representation

The extended classical representation also shares most of the limitations of the classical representation. The main differences are that it does not predetermine the cycle length very strongly (eg, given 4 groups, cycle lengths of 8, 12, 16, etc, weeks are feasible). Hence it also allows for staffing levels on a given weekday to vary over weeks (eg, every 2nd week only half the employees are needed in the night shifts because of maintenance).

Full-by-groups

Full-by-groups does not impose restrictions on the design of rotas that build upon weeks. Still, it requires much space. At the same time it allows a combination with a calendar to ease insight into calendar-specific pros and cons.

Representation by month — hospitals

Representation by month imposes no restrictions on design. Therefore it seems to be more adequate for service industries and hospitals, as changing requirements, high fluctuation, short-term assignments, and different weekly workhours are to be considered. The restrictions imposed by the classical representation do not allow such rotas to be developed.

The only relevant technical restriction is short cycle length. Several rotas cannot be used (eg, the example given in the introduction) due to the limitation of a maximum length. Furthermore, it is difficult to maintain a fair balance of strenuous duties between staff members. An advantage of this representation is that it facilitates taking legal and administrative regulations that are based on month into account.

Assessment of rotas

Obviously, the representations differ substantially in their size. However, the effort to assess rotas varies greatly. In the following discussion we have used the rota presented in figure 2 to illustrate the effort necessary to assess 3 representations (basic series, extended classical, full-by-groups).

All the representations discussed share a drawback. As shifts are represented by letters, it is not immediately visible that a night shift impairs the next day. For example, weekends are strongly impaired by night shifts on Fridays. This limitation could only be overcome by a different representation, in which the position of each shift becomes immediately visible, or by the addition of another symbol (eg, "*") to mark such impaired days off.

The basic series allow immediate assessment of most ergonomic criteria (night shifts in a row, duties in a row, number of consecutive days off, rest hours between

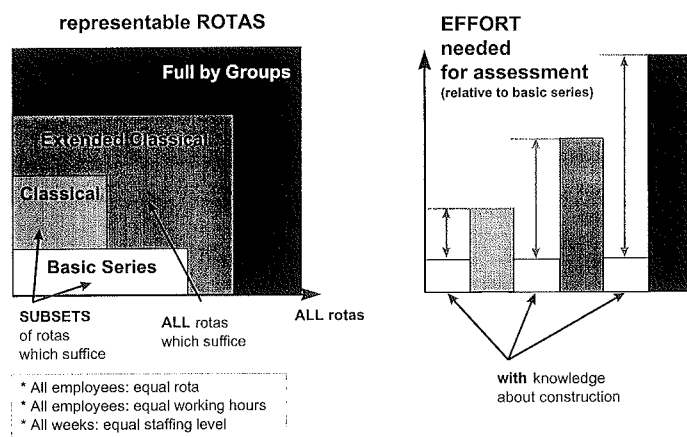


Figure 3. Qualitative description of representable rotas and the effort needed to assess different representations.

duties). Others are easily assessed by simple computations [cycle length, number of days off (number days off = days off in basic series $\cdot 7 \Rightarrow 2 \cdot 7 = 14$), number of blocks of consecutive days off (number blocks off = number blocks off in basic series $\cdot 7 \Rightarrow 1 \cdot 7 = 7$), number of weekends off²]. To assess the impairment by night shifts before the days off, the formulas for assessing basic series can be easily adapted. Each length of a block of days off that is preceded by a night shift is reduced by one. (To continue the example: 7 days are really off (in 8 weeks), no weekend is really free). These computations may not be broadly known, but they are quickly learned. Weekly workhours and the distribution of weekends off over the cycle [discrete mathematics: 8 (length of basic series) = 7 (length of week) + 1; therefore, the weekends completely off and partially off are close to each other] can be roughly assessed too.

The assessment of these criteria in the extended classical and in the full-by-groups representations makes substantial counting necessary. If readers know that the rota of each group is the same, the effort for these 2 representations is somehow similar. The representation full-by-groups is longer, but it makes the reading of only one line necessary (with less line breaks). Still, if the reader is not sure about the equality of the rotas for each group, it is necessary to check equality first.

Given that readers are familiar with basic series and their corresponding computations, basic series are superior to the other representations. The effort for assessment differs roughly by a factor of 7 for the example rota. Still, not all rotas can be represented with basic series, and it is difficult to optimize weekends. The computational advantage of the representation in basic series also declines if readers know that the rota represented in the extended classical or full-by-groups representations is built upon a basic series. Then only the basic series and eventual adaptations (eg, weekends) have to be assessed.

Using an exemplary complexity analysis, we can analyze the rest hours between duties, the number of night shifts in a row, and the rest hours after the night shifts. Let us define the corresponding effort to analyze the basic series of figure 2 as a basic assessment unit. The effort in the other representations is then as follows: If readers know that rotas are built upon a basic series, then 1 unit (effort for basic series) is needed. Otherwise, if readers know that the rota is equal for all groups (by using the extended classical representation of equality analysis), then 7 units are needed; otherwise 4 groups $\cdot 7$ units = 28 units. In this simple example the differences peak at 28 times the effort for a basic series as a maximum. Two issues stand out. First, it is useful to choose the simplest representation capable of covering the design task. (short) basic series are the simplest. Classical and extended classical representations are simpler than a full-by-groups representation. The differences are substantial. Given that the design of rotas in the classical representation is already very complex, a full-by-groups representation is hardly feasible without computer support. Second, knowledge of how the rota was designed may slash the effort to analyze it. In other words, more complex representations can be used if readers know how the rota at hand was developed.

Concluding remarks

The representations discussed in this paper differ strongly in their power to represent all possible rotas. In addition they can differ heavily in the effort needed for the assessment of rotas. This effort again depends on the reader's knowledge of the construction technique applied.

Figure 3 presents different representations and their corresponding representable rotas. On the right side a

² Weekends off = $\sum_{\text{for each block}_i \text{ of days off in the basic series}} \text{length}(\text{block}_i) - 1 \Rightarrow \sum 2 - 1 = 1$.

sketch of the effort for the assessment of typical rotas is given. The effort needed varies with the construction technique applied and with the knowledge of readers about the construction. An exemplary quantitative analysis of the number of rotas representable in each representation is given in the appendix.

Therefore, it is appropriate to use several representations in the design and to change between these representations (eg, basic series for sketches and classical, extended classical, full-by-groups for refinement). Knowledge about the construction of a rota should be maintained. Furthermore, one has to be aware of the limits of each representation to avoid any omission of relevant solutions. Designers of rotas, as well as of computer systems for rota design, should thoroughly consider which representation to choose. They should ideally allow a smooth transition between different representations to grant the exploitation of the benefits of each.

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We want to thank Michael Kundi, Silvia Miksch, and Karin Hörwein for their review of this paper.

Appendix

Number of rotas that can be represented

Simplifying assumptions

Continuous rota should be used with the cycle length in weeks. Each group must do the same number of duties of each kind of shift. On each day, 1 group is assigned to each shift. No other requirements (eg, rest hours) are considered. Unfortunately, it was not possible to develop a closed formula to compute the size of the solution space so far. Therefore a substantially overestimating upper limit of possible rotas (ro) has been used.

Basic series

Let s be the number of shifts per day. The length of the basic series is a multiple (n) of the number of groups (g). If the basic series is written in n columns with the height g , then each row must meet the staffing level requirement. The number of days off in each column (o) is calcu-

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lated as $o = g - s$. Rotas that have the same sequence of shifts but start on different days are eliminated by division by the number of groups. Then the upper limit is $ro = (g!/o!)^n \cdot (1/g)$.

Classical representation

The classical representation can be considered as a specific case of the basic series with $n = 7$. Correspondingly: $ro = (g!/o!)^7 \cdot (1/g)$.

Extended classical representation

The extended classical representation can be considered as a specific case of the basic series with $n = 7 \cdot m$, where m is the number of weeks needed for representing the rota. Correspondingly: $ro = (g!/o!)^{7 \cdot m} \cdot (1/g)$.

Classical

cycle length in weeks	l	5	Groups	g	5
possible rotas	ro	6 E+11	Shifts	s	3

Basic Series

number of rows	n	1	2	3	4	5	6	7	8	9	10
cycle length in weeks	l	5	10	15	20	25	30	35	40	45	50
possible rotas	ro	12	720	43200	2592000	2E+08	9E+09	6E+11	3E+13	2E+15	1E+17
representable in Classical		YES	NO	NO	NO	NO	NO	YES	NO	NO	NO

Extended Classical

represented weeks	m	1	2	3	4	5	6	7	8	9	10
number of rows	n	7	14	21	28	35	42	49	56	63	70
cycle length in weeks	l	5	10	15	20	25	30	35	40	45	50
possible rotas	ro	6 E+11	6 E+11	6 E+11	6 E+11	6 E+11	6 E+11	6 E+11	6 E+11	6 E+11	6 E+11

Full by Groups

represented weeks	m	5	10	15	20	25	30	35	40	45	50
number of cells	n	35	70	105	140	175	210	245	280	315	350
cycle length in weeks	l	5	10	15	20	25	30	35	40	45	50
possible rotas	ro	3 E+61	6 E+123	1 E+186	2 E+248	#	#	#	#	#	#

Figure. Rotas representable in each representation for 5 groups and 3 shifts.*Full-by-groups representation*

The full-by-groups representation can also be considered as a specific case of the basic series with $n = 7 \cdot g \cdot m$ (ie, a much longer rota). Correspondingly:

$$ro = (g!/o!)^{7 \cdot g \cdot m} \cdot (1/g).$$

Comparison

With the use of a maximum cycle of 350 days, the number of rotas shown in the figure above are representable.

Not each rota that is representable in the basic series is also representable in the classical representation. Their number is huge in theory. Practically only short basic series are of relevance. In the case of short basic series their relatively small number makes screening for interesting solutions easier. All basic series can be represented by the extended classical model. All rotas representable by the extended classical model can be represented in the full-by-groups representation. The huge number of possible solutions with the full-by-groups representation makes the need for design strategies obvious.